

# ANALYSIS AND SYNTHESIS EQUATIONS FOR EDGE-COUPLED SUSPENDED SUBSTRATE MICROSTRIP LINE

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## ABSTRACT

The dispersion characteristics of edge-coupled suspended substrate microstrip line is analyzed by using method of lines. Based on the numerical results obtained from this method, a set of closed-form analysis and synthesis equations are presented, valid over a practical application range of structural parameters and substrate dielectric constant. Compared with the data obtained from SUPER COMPACT, the accuracy is found to be within 3.5% for analysis and 5% for synthesis.

## I. INTRODUCTION

The suspended substrate microstrip line is the modified version of microstrip line. Compared with the normal microstrip line, it has some attractive features, such as lower attenuation and larger tolerance of fabrication. Therefore, the suspended substrate microstrip line has been extensively used in millimeter-wave integrated circuits, such as millimeter-wave mixer, oscillator, multiplier and so on. The edge-coupled suspended substrate microstrip line is often used to form such kind of circuits or systems. Several numerical techniques can be used to analyze its dispersion characteristics, in which the spectral-domain technique is the most widely used. But the mathematical procedure is complex and there exists the problem of selecting basis function and

so called "relative convergence". In this paper, the method of lines<sup>(1)-(3)</sup> is used to analyze the dispersion characteristics of such line. The computed results show good agreement with the data obtained from SUPER COMPACT<sup>(4)</sup>. Based on the numerical results, a set of closed-form analysis and synthesis equations are developed by using least-square curve fitting technique. Using these equations, the effective dielectric constant and the characteristic impedances for odd and even mode can be easily calculated. Conversely, the width of strip can be synthesized if the characteristic impedances of odd and even mode are specified. Compared with the data obtained from SUPER COMPACT, the accuracy is found to be within 3.5% for analysis and 5% for synthesis.

## II. ANALYSIS EQUATIONS

The cross sectional view of edge-coupled suspended substrate microstrip line is shown in Fig.1. Considering the practical application range of structural parameters and

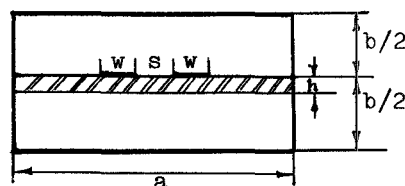


Fig.1 Cross sectional view of edge-coupled suspended substrate microstrip line.

substrate dielectric constant, the developed equations are valid under following conditions

$$\begin{aligned} 2.22 &\leq \epsilon_r \leq 3.8, \\ 0.05 &\leq \frac{w}{a} \leq 0.2, \\ 0.054 &\leq \frac{s}{a} \leq 0.26, \\ 0.4 &\leq \frac{b}{a} \leq 0.85, \\ 0.04 &\leq \frac{h}{a} \leq 0.12. \end{aligned}$$

(1) Effective dielectric constants

A. For odd mode

$$\frac{1}{\sqrt{\epsilon_{ro}}} = 1 - E(\ln \frac{s}{a} - F)(\ln \frac{w}{a} - G) \cdot \ln \epsilon_r \cdot \epsilon_r^{0.03}. \quad (1)$$

where

$$E = 10^{-4} \left\{ 7.7291 \left[ \frac{b}{a} + 6.9924 \left( 9.5 \left( \frac{h}{a} \right)^2 - 1 \right) \right] \left[ \ln \frac{h}{a} + 2.1657 \right] + 561.62 \left( \frac{h}{a} \right)^2 - 6.7772 \left( \frac{h}{a} \right) \right\}. \quad (2a)$$

$$F = 60.4762 \left[ \left( \frac{b}{a} - 6 \frac{h}{a} \right)^2 \left( 1 + 2 \frac{h}{a} \right) - 1.6012 \left( 1 - 8 \left( \frac{h}{a} \right)^2 \right) \right] \left( e^{\frac{h}{a}} - 1.04634 \right) + 1721.61 \left( \frac{h}{a} \right)^2 + 19.027 \left( \frac{h}{a} \right). \quad (2b)$$

$$G = 0.05154 \left[ - \left( \frac{b}{a} - 0.3 \right)^2 + 0.3653 \left( 1 - 3 \left( \frac{h}{a} \right)^2 \right) \right] \left[ e^{\frac{h}{a}} + 54.554 \right] + 6551.53 \left( \frac{h}{a} \right)^3 + 34.76 \left( \frac{h}{a} \right). \quad (2c)$$

Compared with the data obtained from SUPER COMPACT, the accuracy is found to be within 2%.

B. For even mode

$$\frac{1}{\sqrt{\epsilon_{re}}} = 1 - \left( D \frac{s}{a} + H \cdot \ln \left( \frac{w}{a} \right) + R \right) (\ln \epsilon_r) \epsilon_r^{0.06} \left( \frac{\epsilon_r}{2.3} \right)^{\left( \frac{b}{a} - 0.4 \right) / 3}. \quad (3)$$

where

$$D = k_{11} \left( \frac{h}{a} \right)^3 + k_{12} \left( \frac{h}{a} \right)^2 + k_{13} \left( \frac{h}{a} \right) + k_{14}. \quad (4)$$

$$k_{11} = -23.794 \left( \frac{b}{a} \right)^2 + 29.953 \left( \frac{b}{a} \right) + 52.861. \quad (5a)$$

$$k_{12} = 0.92658 \left( \frac{b}{a} \right)^2 + 3.2713 \left( \frac{b}{a} \right) - 20.653. \quad (5b)$$

$$k_{13} = -1.5075 \left( \frac{b}{a} \right)^2 + 1.3914 \left( \frac{b}{a} \right) + 0.75773. \quad (5c)$$

$$k_{14} = -0.25477 \left( \frac{b}{a} \right)^3 + 0.60430 \left( \frac{b}{a} \right)^2 - 0.48538 \left( \frac{b}{a} \right)$$

$$+ 0.16713. \quad (5d)$$

$$H = k_{21} \left( \frac{h}{a} \right)^3 + k_{22} \left( \frac{h}{a} \right)^2 + k_{23} \left( \frac{h}{a} \right) + k_{24}. \quad (6)$$

$$k_{21} = -11.459 \left( \frac{b}{a} \right)^2 + 18.744 \left( \frac{b}{a} \right) - 17.109. \quad (7a)$$

$$k_{22} = 7.4499 \left( \frac{b}{a} \right)^2 - 11.927 \left( \frac{b}{a} \right) + 7.4606. \quad (7b)$$

$$k_{23} = -0.33432 \left( \frac{b}{a} \right)^2 + 0.552 \left( \frac{b}{a} \right) - 0.37211. \quad (7c)$$

$$k_{24} = 0.014317 \left( \frac{b}{a} \right)^2 + 0.030418 \left( \frac{b}{a} \right) - 0.0314 \quad (7d)$$

$$R = k_{31} \left( \frac{h}{a} \right)^2 + k_{32} \left( \frac{h}{a} \right) + k_{33}. \quad (8)$$

$$k_{31} = -48.729 \left( \frac{b}{a} \right)^3 + 115.45 \left( \frac{b}{a} \right)^2 - 92.939 \left( \frac{b}{a} \right) + 22.5953. \quad (9a)$$

$$k_{32} = 1.4237 \ln \left( \frac{b}{a} + 3.2411 \right) - 0.18519. \quad (9b)$$

$$k_{33} = 0.014478 \ln \left( \frac{b}{a} + 0.23568 \right) - 0.024308. \quad (9c)$$

Compared with the data obtained from SUPER COMPACT, the accuracy is found to be within 2.5%.

(2) Characteristic impedances

$$\text{For odd mode, } Z_{oo} = Z_o / \sqrt{\epsilon_{ro}}; \quad (10)$$

$$\text{For even mode, } Z_{oe} = Z_e / \sqrt{\epsilon_{re}}.$$

Where  $Z_o$  and  $Z_e$  are characteristic impedances of odd and even mode, respectively, when dielectric constant  $\epsilon_r = 1$ .

A. For odd mode

$$Z_o = 60\pi^2 / \ln \left( 2 \cdot \frac{1 + \sqrt{k'_1}}{1 - \sqrt{k'_1}} \right) \quad (11)$$

Where

$$k'_1 = \sqrt{1 - k_1^2}, \quad k_1 = \frac{\text{tg } \varphi'_2}{\text{tg } \varphi'_1}.$$

$$\left. \begin{aligned} \varphi'_1 &= Q_1 \cdot \text{th} \left( \frac{2w+s}{b} Q_2 \right) + Q_3, \\ \varphi'_2 &= Q_1 \cdot \text{th} \left( \frac{s}{b} Q_2 \right) + Q_3. \end{aligned} \right\} \quad (12)$$

$$Q_1 = 0.0361168 \exp(5.12k) + 1.4404$$

$$Q_2 = -0.0316177 \exp(3.947k) + 1.07319.$$

$$Q_3 = 0.15988k^3 - 0.0895k^2 + 0.02535k + 0.002311.$$

$$k = \sqrt{1 - \left( \frac{e^{\pi a/b} - 2}{e^{\pi a/b} + 2} \right)^4}$$

Compared with the data obtained from SUPER COMPACT, the accuracy is found to be within 3%.

B. For even mode

$$Z_e = 60\pi^2 / \ln(2 \frac{1+\sqrt{k'_2}}{1-\sqrt{k'_2}}). \quad (13)$$

Where

$$k'_2 = \sqrt{1 - k_2^2}.$$

$$k_2 = \sqrt{\frac{1 + tg^2 \phi'_2}{1 - tg^2 \phi'_1}}.$$

$\phi'_1$  and  $\phi'_2$  are same as Eq.(12).

### III. SYNTHESIS EQUATIONS

Let  $p = \frac{Z_{oe} - Z_{oo}}{Z_{oe} + Z_{oo}}$ ,  $\frac{s}{a}$  is given,

$$p_1 = Z_{oe} / Z_{oo} = \frac{1+p}{1-p},$$

then

$$\frac{w}{a} = \exp\left(\frac{t_2 + \sqrt{t_2^2 - 4t_1 t_2}}{2t_1}\right) \quad (14)$$

where

$$t_1 = HB_1 \left( \frac{\epsilon_r}{2.8} \right)^{\frac{b/a-0.4}{3}} / \epsilon_r^{0.02}.$$

$$t_2 = B_1 + p_1 \cdot E(\ln \frac{s}{a} - F) - [B_1 (D \frac{s}{a} + R) + HB_2]$$

$$\left( \frac{\epsilon_r}{2.8} \right)^{\frac{b/a-0.4}{3}} / \epsilon_r^{0.02}.$$

$$t_3 = p_1 - B_2 + p_1 \cdot G \cdot E(\ln \frac{s}{a} - F) + B_2 (D \frac{s}{a} + R)$$

$$\left( \frac{\epsilon_r}{2.8} \right)^{\frac{b/a-0.4}{3}} / \epsilon_r^{0.02}.$$

$$B_1 = d_1 \left( \frac{s}{a} \right)^4 + d_2 \left( \frac{s}{a} \right)^3 + d_3 \left( \frac{s}{a} \right)^2 + d_4 \left( \frac{s}{a} \right) + d_5.$$

$$d_1 = 49228.93 \left( \frac{b}{a} \right)^3 - 86515.93 \left( \frac{b}{a} \right)^2 + 50484.33 \left( \frac{b}{a} \right) - 9992.325.$$

$$d_2 = -29055.10 \left( \frac{b}{a} \right)^3 + 51037.57 \left( \frac{b}{a} \right)^2 - 29730.67 \left( \frac{b}{a} \right) + 5949.112.$$

$$d_3 = 5949.73 \left( \frac{b}{a} \right)^3 - 10436.34 \left( \frac{b}{a} \right)^2 + 6087.17 \left( \frac{b}{a} \right) - 1236.955.$$

$$d_4 = -486.14 \left( \frac{b}{a} \right)^3 + 849.65 \left( \frac{b}{a} \right)^2 - 495.616 \left( \frac{b}{a} \right) + 104.9031.$$

$$d_5 = 11.9172 \left( \frac{b}{a} \right)^3 - 20.3952 \left( \frac{b}{a} \right)^2 + 11.8292 \left( \frac{b}{a} \right) - 2.8853.$$

$$B_2 = m_1 \ln \left( \frac{s}{a} + m_2 \right) + m_3.$$

$$m_1 = 16.132 \left( \frac{b}{a} \right)^3 - 27.573 \left( \frac{b}{a} \right)^2 + 14.6033 \left( \frac{b}{a} \right) - 2.74812.$$

$$m_2 = -15.443 \left( \frac{b}{a} \right)^3 + 27.658 \left( \frac{b}{a} \right)^2 - 16.409 \left( \frac{b}{a} \right) + 3.2731.$$

$$m_3 = 12.9051 \left( \frac{b}{a} \right)^3 - 20.6769 \left( \frac{b}{a} \right)^2 + 10.0935 \left( \frac{b}{a} \right) - 0.76533.$$

Where E, F, G, D, H and R are given in Eqs.(2), (4), (6) and (8).

Compared with the data obtained from SUPER COMPACT , the accuracy is found to be within 5%.

#### CONCLUSION

Based on the numerical results obtained by method of lines, a set of closed-form equations for analysis and synthesis of edge-coupled suspended substrate microstrip line are presented. Using these equations, the computer aided design of such transmission line can be carried out easily.

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